Stability of Yang-Mills-Higgs field system in the homogeneous self-dual vacuum field

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Abstract

Classical dynamics of SU(2) model gauge field system with Higgs field is considered in the homogeneous nonperturbative self-dual vacuum field. The regions of instability in parametric space are detected and described analytically.

Introduction

Much attention has been paid in the last decade to chaos in quantum field theory. At first, non-abelian Yang-Mills gauge fields were investigated without spontaneous symmetry breaking. It was shown analytically and numerically that classical Yang-Mills theories are inherently chaotic ones [1, 2]. Further research of spatially homogeneous field configurations [3] showed that spontaneous symmetry breakdown leads to order-chaos transition at some density of energy of classical gauge fields [4, 5], while dynamics of gauge fields in the absence of spontaneous symmetry breakdown is chaotic at any density of energy. Study of chaos in classical gauge Yang-Mills theories gives an opportunity to understand the influence of Higgs fields on chaotic dynamics of gauge field system. It was demonstrated that classical Higgs fields regularize chaotic dynamics of classical gauge fields at densities of energy less than critical and lead to appearance of order-chaos transition [4]. Chaos in Yang-Mills fields is also considered in connection with confinement [6].

It was shown [7] that quantum fluctuations of vector gauge fields in $SU(2) \bigotimes U(1)$ theory lead to regularization the dynamics of gauge fields at low densities of energy, and order-chaos transition occurs with rise of energy density of gauge fields. It was also observed that if the ratio of coupling constants of Yang-Mills and Higgs fields is larger than some critical value, quantum corrections do not affect the chaotic dynamics of Yang-Mills and Higgs fields. It was demonstrated that centrifugal term in the model Hamiltonian increases the region of regular dynamics of Yang-Mills and Higgs fields system at low densities of energy [8].

The system of Yang-Mills and Higgs fields has an infinite number of degrees of freedom and it is too complicated to be investigated directly. In order to reduce the number of degrees of freedom, following other authors, we consider only spatially homogeneous fields. This model is a particular case of the general one.

Spatially homogeneous field models allow one to investigate the main properties of inhomogeneous fields.

In this paper classical dynamics of SU(2) model gauge field system with Higgs field is considered in the homogeneous self-dual vacuum field. Various properties of this field in SU(2) gauge theory were investigated originall in [9, 10, 11, 12]. It was demonstrated that self-dual homogeneous field provides the Wilson confinement criterion [13]. Therefore this field is at least a possible source of confinement in QCD if it can be shown that such a field is a dominant configuration in the QCD functional integral [13, 14].

Our model consists of homogeneous perturbative and nonperturbative Yang-Mills field and the Higgs fields. In this paper we investigate the influence of nonperturbative homogeneous self-dual field on chaotic dynamics of Yang-Mills and Higgs fields. We demonstrate that dynamics of our system depends on the parameters of the model. Stable and chaotic regions in parametric space are detected. Their bounds are described analytically.

1 Homogeneous self-dual field

Homogeneous self-dual field is characterized by the next expressions [10, 11, 13]:

$$B_\mu^a = \frac{1}{2} n^a B_{\mu\nu} x_\nu, \quad B_{\mu\nu} = -B_{\nu\mu}$$

$$B_{\mu\nu} B_{\mu\rho} = B^2 \delta_{\nu\rho}, \quad B = const$$

$$\widetilde{B}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} B_{\alpha\beta} = \pm B_{\mu\nu}$$

$$B_{ij} = -\epsilon_{ijk} B_k, \quad B_{j4} = \pm B_j.$$

The positive and the negative signs in the last two lines correspond to the self-dual and anti-self-dual fields respectively. The color vector n^a points in some fixed direction which can be chosen as $(n^1, n^2, n^3) = (0, 0, 1)$ [10, 11].

2 Model potential of the system

The Lagrangian of SU(2) gauge theory with the Higgs field in Euclidean metrics is

$$L = -\frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a - \frac{1}{2} (D_\mu \phi)^+ (D_\mu \phi) - V(\phi),$$

where ϕ is the real triplet of the Higgs scalar fields. Classical potential of Higgs fields has the following form

$$V(\phi) = \mu^2 \phi^2 + \lambda \phi^4,$$

The Yang-Mills field tensor is:

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g \epsilon^{abc} A_\mu^b A_\nu^c,$$

and

$$D_\mu \phi = \partial_\mu \phi - igA^b_\mu T^b \phi,$$

where $A^a, \quad a = 1, 2, 3$ are the three non-abelian Yang-Mills fields and T^a are the generators of the group SU(2) in adjoint representation, g denotes the coupling constant of non-abelian gauge fields.

The vacuum in our model is realized by the homogeneous self-dual field. This field represents the nonperturbative component of Yang-Mills field potential. We regard self-dual field as an external field. It is taken into account by substituting modified vector potential in the Yang-Mills-Higgs Lagrangian

$$A^a_\mu \to A^a_\mu + B^a_\mu$$

where A^a_μ is the perturbative component and B^a_μ is the nonperturbative component of the Yang-Mills field.

We work in the gauge:

$$A^a_0 = 0,$$

and consider spatially homogeneous field configurations [4]

$$\partial_i A^a_\mu = 0, \qquad \partial_i \phi^a = 0, \qquad i = 1..3.$$

When $\mu^2 < 0$ the Higgs potential $V(\phi)$ has a minimum at non-zero ϕ:

$$|\underline{\phi_0}| = \sqrt{\frac{-\mu^2}{4\lambda}} = \upsilon,$$

This Higgs vacuum is degenerate and after spontaneous symmetry breaking it can be chosen as

$$\underline{\phi_0} = (\phi_1, \phi_2, \phi_3) = (0, 0, \upsilon).$$

The direction of the nonperturbative field can be chosen arbitrarily. We will assume that it is directed along Z axis [10, 11, 13]. The tensor $B_{\mu\nu}$ will be the following:

$$B_{\mu\nu} = \begin{pmatrix} 0 & -B & 0 & 0 \\ B & 0 & 0 & 0 \\ 0 & 0 & 0 & \pm B \\ 0 & 0 & \mp B & 0 \end{pmatrix}$$

It can be also rewritten in the following form

$$\vec{B} = (B_1, B_2, B_3) = (0, 0, B).$$

In this work we consider small perturbative field A^a_μ on the background of the nonperturbative one. We have retained terms up to the second order in A^a_μ [11]. In this approximation the potential of the perturbative field

$$V_A = \frac{1}{4} g^2 \left\{ (\vec{A}^a)^2 (\vec{A}^b)^2 - (\vec{A}^a \vec{A}^b)^2 \right\}.$$

is neglected.

If $A_1^1 = q_1$, $A_2^2 = q_2$ and the other components of the perturbative Yang-Mills fields are equal to zero, the potential of the model is:

$$V = B^2 + \frac{1}{2} g^2 \upsilon^2 (q_1^2 + q_2^2) + gBq_1q_2 + \tag{1}$$

$$+\frac{1}{8} g^2 B^2 \left\{ q_1^2 (x^2 + z^2 - t^2) + q_2^2 (y^2 + z^2 - t^2) \right\},$$

where x,y,z are the spatial coordinates and t - time.

3 Stability of the model

Stability of the model is investigated using well known technique based on the Toda criterion of local instability [15]. The sign of the Gaussian curvature determines whether the system is stable. The positive curvature indicates stable dynamics, negative - chaotic.

The Gaussian curvature for potential (1) has the following form

$$K_G = C_1 C_2 - g^2 B^2, \tag{2}$$

where

$$C_1 = g^2 \left[\upsilon^2 + \frac{1}{4} B^2 (x^2 + z^2 - t^2) \right],$$

$$C_2 = g^2 \left[\upsilon^2 + \frac{1}{4} B^2 (y^2 + z^2 - t^2) \right].$$

It can be seen from the last expressions that the stability of the model depends only on the set of parameters and coordinates. The model has three parameters and four coordinates: the self-coupling constant g, the value of the nonperturbative field B, the value of the Higgs field υ, three spatial coordinates x,y,z and the time t. Therefore one can regard the seven-dimensional space of parameters and coordinates values. Every point in this space corresponds to the particular set of the parameters and coordinates. This set determines the stability of the model. Thus one can associate the point in the regarded space with the stability of the model. Therefore the stability and instability regions are presented in this space. The expressions for bound surfaces in the seven-dimensional space between the stability and instability regions demonstrate the spacing of these regions. These surfaces can be obtained as the manifolds where the Gaussian curvature (2) is equal to zero.

Different values of spatial coordinates x,y and z correspond to different types of the potential. Our model has a distinguished point ($x = y = z = 0$). If $x = y = 0$ the potential changes along Z axis. This type of potential will be investigated in subsection 3.1. In subsection 3.2 we consider the potential in an arbitrary point (1).

3.1 Stability of the model along Z axis

The potential on Z axis is obtained from (1) by setting x and y to zero:

$$x = 0, \quad y = 0.$$

Thus (1) transforms to the next form

$$V = B^2 + \frac{1}{2}g^2 \left\{ \upsilon^2 + \frac{1}{4}B^2(z^2 - t^2) \right\} (q_1^2 + q_2^2) + gBq_1q_2 \qquad (3)$$

In this case we have five parameters.The Gaussian curvature (2) is

$$K_G = g^4 \left[\upsilon^2 + \frac{1}{4}B^2(z^2 - t^2) \right]^2 - g^2B^2.$$

The roots of this expression correspond to the bound surfaces between stability and instability regions and have the following form:

$$(t^2)_\pm = 4\frac{\upsilon^2}{B^2} + z^2 \pm 4\frac{1}{gB}. \qquad (4)$$

These expressions describe two four-dimensional surfaces in five-dimensional space of parameters and coordinates values. Analysis of the expression for the Gaussian curvature showed that the instability region is situated between these bounds.

The expression

$$\Delta(t^2) = 8\frac{1}{gB} \qquad (5)$$

describes the width of the instability region.

It can be seen that in the case of the distinguished point (z=0) the location of instability region depends on two parameters - the values of the Higgs and non-perturbative fields. In the case of an arbitrary point on Z axis the location of the instability region is shifted along time axis in comparison with the distinguished point according to the expression (4). The width (5) of this region remains unchanged. This case is illustrated on the first figure (Fig.1).

Two bound lines (4) are plotted on the figures. The instability region is situated between them.

The width of the instability region (5) depends on the value of the nonperturbative field and the self-coupling constant. The first is demonstrated on the first figure (Fig.1). One can notice that the self-coupling constant affects the width of the instability region only (Fig.2).

The potential energy (3) decreases with time. The density of energy which corresponds to the chaos to order transition is:

$$V_+ = B^2 - \frac{1}{2}gB(q_1 - q_2)^2.$$

The sign '+' means that this transition occurs at time $(t)_+$.

3.2 Stability of the model in the arbitrary point

Now let us investigate the whole form of the potential (1). In this case the potential depends on the point in the seven-dimensional space of parameters and coordinates

values. The Gaussian curvature takes the general form (2). The expression for bound surfaces reads:

$$(t^2)_\pm = 4\frac{v^2}{B^2} + z^2 + \frac{1}{2}x^2 + \frac{1}{2}y^2 \pm \pm 4\sqrt{\frac{1}{g^2B^2} + \frac{1}{2^6}(x^2 - y^2)^2}. \quad (6)$$

The region under investigation is shifted along the axis T in comparison with the case of the third axis. This shift is proportional to the square of the coordinates x and y.

Similar to the case of the third axis we obtain the expression for the width of the instability region:

$$\Delta(t^2) = 8\sqrt{\frac{1}{g^2B^2} + \frac{1}{2^6}(x^2 - y^2)^2}. \quad (7)$$

This expression is more complicated than the previous one. It coincides with (5) when $x = y$. In other points it increases strongly with the difference between x and y (Fig.3).

Two bound lines (6) are plotted on the figures. The instability region is situated between them.

The coupling affects the width of the instability region when $x = y$. Otherwise the width of the instability region mainly depends on the difference between x and y (Fig.4).

It can be seen from this figures that the type of stability is instantaneously changed on the lines which are parallel to the axes X and Y. It happens in the critical time which reads

$$(t^2)_d = 4\frac{v^2}{B^2} + z^2 + u^2,$$

where $u = x$ or $u = y$ if the line is parallel to the axis X or Y respectively. The direction of order-to-chaos transition depends on the correlation between the coordinates x and y. The example is demonstrated on the Fig.3, where in the points with $x < y$ chaos-order transition occurs, while the points with $y < x$ correspond to order-chaos transition. Discontinuously stability change is'nt occurred on the lines which are non-parallel to the axis X or Y.

Conclusions

Classical dynamics of SU(2) model gauge field system with Higgs field was considered in the homogeneous self-dual vacuum field. The stability of the model was investigated based on the Toda criterion of local instability. Analysis showed that the stability of the model depends on the set of the parameters and coordinates. The value of perturbative field does not influence on the model stability in our approximation. Model has three parameters and four coordinates: the self-coupling constant g, the value of the nonperturbative field B, the value of the Higgs field v, three spatial coordinates x, y, z and time t. Therefore one can associate the point

in the seven dimensional space with the stability of the model. The stability and instability regions were detected. The analytical expressions for the bounds of the regions were obtained for the points on Z axis ($x = y = 0$) and for an arbitrary point.

The moments of time of appearance and disappearance of the instability region present in model with arbitrary set of parameters and coordinates. The time of existence of the instability region tends to infinity at small nonzero values of non-perturbative field (Fig.1, Fig.2). This behavior could be connected with confinement [6, 13, 16].

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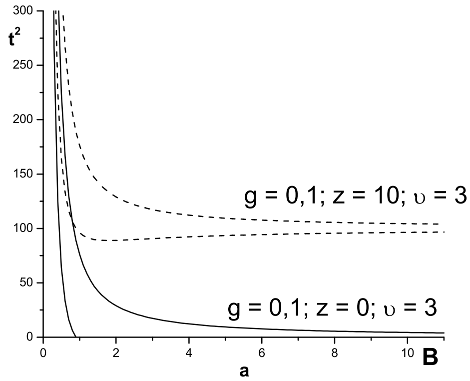
Figures



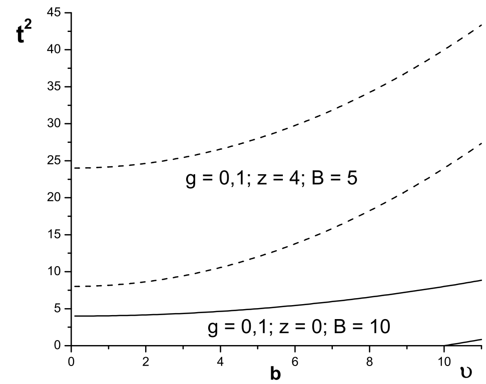


Figure 1: The instability region for points on the axis Z (x = y = 0) in plane **a)** (t^2 - B) and **b)** (t^2 - υ)

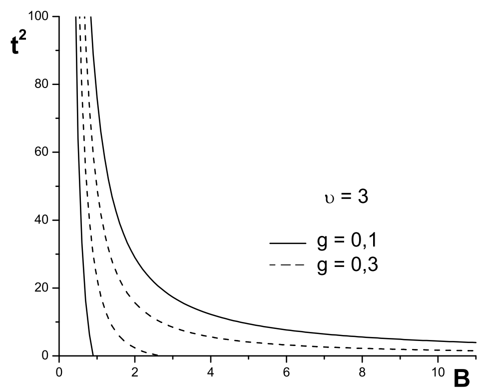


Figure 2: The instability region in plane (t^2 - B) for distinguished point (x = y = z = 0)

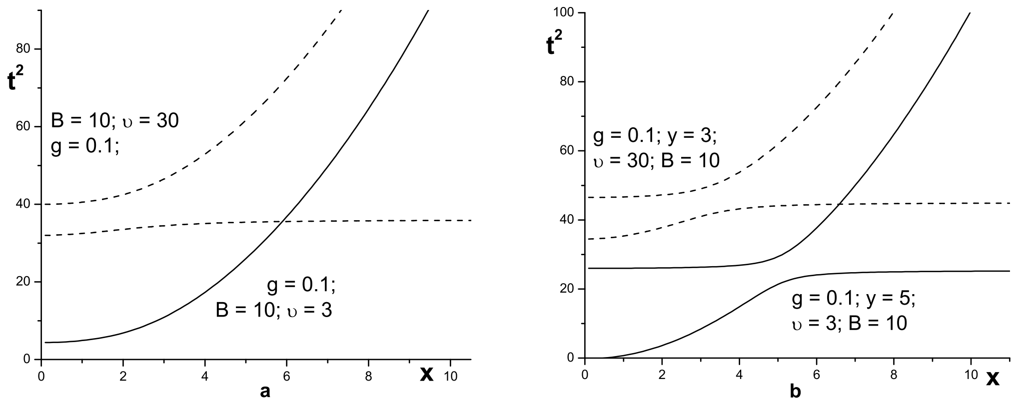


Figure 3: The instability region in plane (t^2 - x) for points **a)** on the axis X (y = z = 0) and **b)** on the line which is parallel to axis X (z = 0)

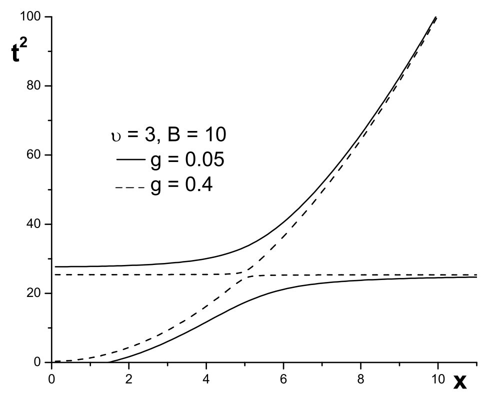


Figure 4: The instability region in plane (t^2 - x) for points on the line which is parallel to axis X (y = 5, z = 0)